Practical application of Moment of Inertia (2)

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In part 1, I showed how to get Moment of Inertia (MoI) value for each part, hammer, wippen, key and the linked MoI at the key. This month, all required data is entered into spread sheets and you will see how to set the parameter to get the best possible combination between static balancing (balance weight) and kinetic resistance (the MoI) for your purpose.

How we can change the Moment of Inertia, possibilities and limits.

From the calculations and sample results from part 1, we know the MoI of the hammer is the biggest contributor among the three components of the action, hammer, wippen and key. The hammer however presents us with limited possibilities to adjust its weight. Because it has only a small wooden moulding, we can likely only reduce or increase its weight by **a maximum** of 1 gram after normal tapering, arcing and tail shaping. We can make only small reductions in the mass of hammers by more aggressive tapering, arcing, tailing and by boring small holes in the moulding. The ability to increase hammer mass is also limited. We can put small leads in to hammer moulding but we have very little room to do it.

Changing the hammer weight of our sample C4 from the Steinway D, by 1 gram makes a change of 18,632 gcm^2 of MoI at the key (1g x 13^2 x 10.5^2). A 1 gram change would adjust the hammer weight of about 10%, and result in a change of the MoI by about the same amount. (It would also make a change of about 5.5 grams to the balance weight, i.e. 1 gram x 5.5, the strike ratio).

This amount is large enough to change the MoI and balance weight quite effectively, however other factors must be considered. One is the tonal quality of the note. If the mass of the hammer is reduced too much, it may not produce enough volume and quality. Another point to consider is the evenness of the hammer strike weight in the piano. As Stanwood suggests, an evenly curved hammer strike weight will give a smooth transition of tonal quality as well as an even touch weight. Recognising this, a practical limit to hammer weight modification might be 0.7 grams. Some hammers may be able to be reduced by that amount and others may only be able to be modified some 0.2 grams. So the change in the hammer MoI at the key could be between 3,700 and 11,200 gcm^2.

Conversely, the key stick has very high MoI values and many possibilities for modifications, the keys don’t exert a large influence on the MoI of the whole action, but making modifications will alter the MoI. As an example of achieving the least possible inertia in a key the MoI in (Fig. 12) has been reduced about 9,200 gcm^2 by relocating key leads, this is a considerable amount, it would equate to a hammer weight reduction of 0.5grams.

As the wippen doesn’t have a significant effect on either the balance weight or MoI, we are not including them in these discussions.

Fig. 8 shows possible modifications to C4 of our Steinway D. In this example, hammer mass was reduced by 0.4 grams, one key lead was relocated and the balance punching was cut and moved to the rear side of the balance pin. The key balance weight became 38 grams. The MoI of the whole action felt at the key was reduced 8% (of this 72 % from the hammer and 27 % from the key). This key felt much lighter than the original because the balance weight was reduced from 40 grams to 38 grams, and 8% was removed from the MoI.

MoI values, original and after modification

		Moment of Inertia values						contribution at key from		
		Hammer	Hammer at key	Wippen	Wippen at key	Key	Whole action at key	Hammer	Whippen	Key
Predicted MoI after modification		1,775	187,516	756	4,162	44,657	236,336	15,061	169	5,745
% deducted from original		4%		0%		11%	8%	72%	1%	27%
Original MoI (gcm^2)		1,842	202,577	756	4,332	50,403	257,311			
Hammer Strike Weight	original	10.9 grams								
	after modified	10.5 grams								
Hammer distance (cm)	original	13.0 cm								

L_{KO}: the distance between the pivot of the key and the top of the capstan screw

15.2 cm   14.9 cm (after cutting punching cloth)

L_{WI}: the distance between the pivot of the wippen and the center bottom of the wippen heel

6.35 cm

L_{WO}: the distance between pivot of the wippen and the contact point, jack to hammer knuckle

9.2 cm

L_{HI}: the distance between contact point, the knuckle to jack and the pivot of the shank flange

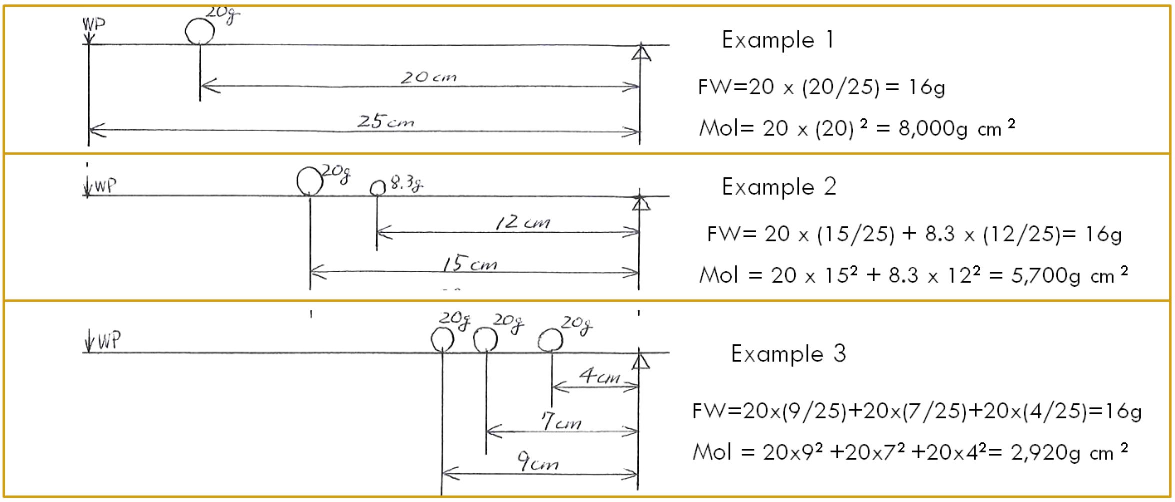
2.1 cm

(Figure 8) Reducing the MoI by changing the mass of hammer and relocating key lead

Nakamura front weight spread sheet and Moment of Inertia spread sheet

My method of calculation has two merits. It is technician friendly. If you have reasonable math and spread sheet skills, you will enjoy the benefits. This method doesn't require measuring angular acceleration and torque which require complex scientific equipment and analysis.

Another merit of this method is that it works in conjunction with the Stanwood equation chart. Using these charts (Fig. 11, 12 and 13) lets us analyze possible touch settings on the computer by combining static balancing (Stanwood) and the kinetic movement of the action. Let me explain.



- ❑WP: Weighting Position i.e. 13 mm from front edge of the key.
- ❑ FW=Mass of lead x (L to lead)/(L to WP)

(Fig. 9) Front weight and the MoI of model key to compare different leading patterns

Positioning the key leads affects both front weight and MoI. See Fig. 9. There are 3 different patterns of key leading here. Imagine that the only mass the key has is the key lead, no mass otherwise. All the samples have same front weight of 16g. But they have very different amounts of MoI.

Front weight can be calculated by the mass of lead multiplied by the distance between pivot and mass center of the lead divided by the distance between pivot and weighting position as shown above.

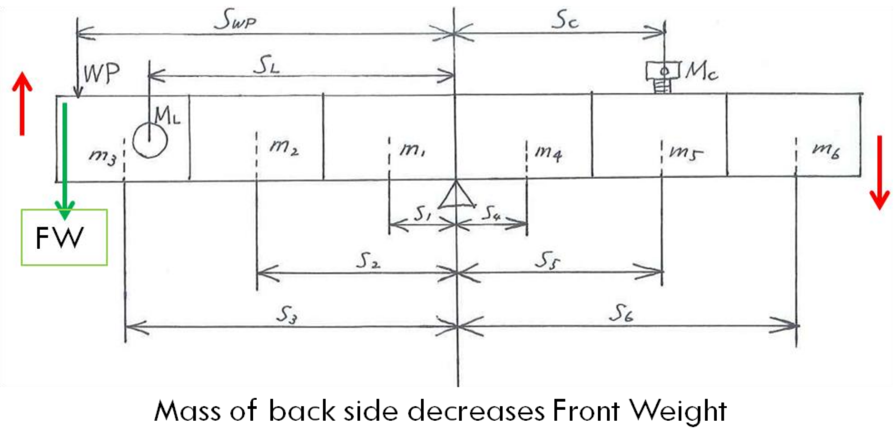
Ex. 1 has 16g of front weight and one big lead near the front end of the key. The MoI of this key is 8,000 gcm^2

Ex. 2 has the same 16g of front weight but two leads in the middle of the front side of the key. The MoI is 5,700 gcm^2. Ex. 2 has 29% less MoI than Ex.1 even though it has the same front weight.

Ex. 3 has the same 16g of front weight and three bigger leads nearer to the balance rail. The MoI is 2,920 gcm^2. Ex. 3 has 64% less MoI than Ex. 1.

The touch feeling is different when the leading patterns are different even if the front weight is the same. Of these samples, example 3 will feel much lighter than example 1 because the MoI is much smaller, even though the mass of the key is greater.

In an actual key, the mass in the back side of the key must also be considered. The mass of the back side of the key reduces front weight as it counter balances the front side, (see Fig. 10).



(Fig. 10) Simplified model of key stick explaining counter balancing

The mass the back side of the key however adds to the MoI of the whole key because the whole key rotates as one object at the balance pin. So we get two different values from MoI spread sheet, MoI value and front weight.

Nakamura FW spread sheet. See Fig. 11. Measured original front weight is shown at the top of the chart. This is actual measurement using the Stanwood system. Calculated original front weight is shown next. This is the value calculated by Nakamura MoI spread sheet. The difference in values doesn't matter at this stage, we will account for it later in the calculation.

Nakamura Front Weight spread sheet

	40	C4	measured original FW	28.9						
A	40	C4	calculated original FW	26.7	front	49.4	back	22.7	difference	2.2
B			least inertia with new FW	22.7	front	45.4	back	22.7	desired FW	22.7
C			practical setting with new FW	22.7	front	45.4	back	22.7	desired FW	22.7
D			key stick only FW	5.8	front	28.5	back	22.7		

(Fig. 11) Nakamura FW spread sheet

Using the Nakamura spread sheets and Stanwood equation spread sheet to analyze the action

Using the three charts, **Nakamura FW spread sheet** (Fig. 11), **Stanwood Equation spread sheet** (Fig. 12) and **Nakamura MoI spread sheet** (Fig. 13); we can analyze the touch of the action and if necessary change the parameters before making any physical modifications. Analyzing both static balance and kinetic resistance will give us a wider view. In the example discussed below the pianist described the touch as a little heavy and slightly slow. My objective was to reduce both weight using the Stanwood spread sheet and then MoI spread sheet.

First, look at Stanwood's equation spread sheet (Fig. 12).

Stanwood Equation spread sheet

						Front	(BW+FW)		=			Back	(WBW+HSWxR)			FW	HSW
	Key #	Note	details specified	DW	UW	BW	F	FW		KR	WW	WBW		HSW	R	ceiling	index
a	40	C4	original	51	29	40.0	11.0	28.9		0.53	17.1	9.06		10.9	5.49	30.0	#10
b			possible HSW adjustment	49	27	38.0	11.0	28.9		0.53	17.1	9.06		10.5	5.51		#9
c			possible punching cut	45	23	34.0	11.0	28.9		0.53	17.7	9.38		10.5	5.10		
d			possible releading	49	27	38.0	11.0	24.9		0.53	17.7	9.38		10.5	5.10		

(Figure 12) Stanwood's equation spread sheet (Stanwood 2000)

We analyzed the possible refinement or adjustment of hammer strike weight, strike ratio and balance weight by using the Stanwood spread sheet. For example, the first row of the chart shows the original data for the note. It has 40g balance weight, 28.9g front weight and a 5.5 strike ratio. 28.9g front weight is slightly lighter than Stanwood's front weight ceiling which is 30g for this note.

I worked through the following steps to see what weight I could reduce.

A. If we could reduce the HSW by 0.4 grams, the balance weight will decrease by about 2 grams (0.4g x 5.5). Reducing the hammer weight affects only the balance weight. To predict the effect of the hammer weight modification on the balance weight, you will need to change the down weight and up weight values in the spread sheet while maintaining the friction and strike ratio values.

B. Next, to reduce the action strike ratio to further lighten the touch, Stanwood suggests we can cut the balance punching cloth in half and place it at the back side of the balance pin. This now almost common practice will reduce the strike ratio by around 0.4.

C. In steps A and B we achieved reduced HSW and BW. The BW is now 34 grams, this is too low. To achieve our desired balance weight of 38 grams, we need to modify the key weighting. **Explanation.** This is because when re-leading, we only change the left side of equation. We don’t change the wippen, hammer and strike ratio, so the right side of the equation stays the same. Two items on the left side of the equation change. If we put leads into front of the key, the balance weight decreases and front weight increases by the same amount. If we take leads out of the front side of the key, the balance weight increases and front weight decreases by the same amount.

We will achieve 24.9 grams front weight by making these changes. The Stanwood spread sheet allows us to know this without having to make any physical changes to the action or key.

Analyzing the Moment of Inertia.

Look at Nakamura FW spread sheet (Fig. 11) again. The original measured front weight shows top row, 28.9 grams. The calculated front weight from the MoI spread sheet is shown second row as 26.7 grams. We need to bring this 2.2 gram discrepancy into our calculation. Our desired measured front weight is 24.9 grams, so our desired calculated front weight is 22.7 grams, (22.7 + 2.2= 24.9).

Rows B and C in FW spread sheet (Fig 11) are the calculated values taken from rows B and C of MoI spread sheet (Fig13.) which can be achieved with different leading patterns.

								Lead (front)									
	Key #	Note	Status	Whole Inertia	FW point dstnc	Center of torque	CoG position ratio	distance #5 (mm)	mass of lead	distance #4 (mm)	mass of lead	distance #3 (mm)	mass of lead	distance #2 (mm)	mass of lead	distance #1 (mm)	mass of lead
A	40	C4	Original	50403	292	222.8	0.731	244	14.5	220	8.6	157	4.3				
B			least inertia	41181	292	84.7	0.278					119	19.4	85	19.4	50	19.4
C			practical setting	44657	292	152.8	0.501			220	8.6	157	4.3	122	19.4		
D			key stick only	36548	292												

(Figure 13) Nakamura MoI (key) chart

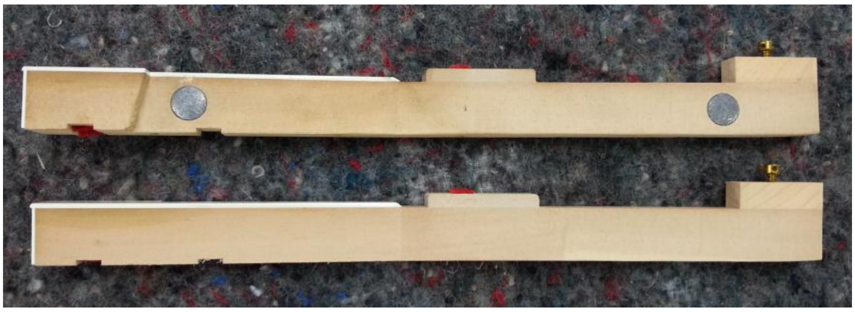
Row B has a leading pattern with the least MoI value. 3 pieces of 15 mm leads are installed at shown distances from the balance pin. This predicts 22.7 grams of front weight and 41,200 gcm^2 of MoI. This sample calculation gives a front weight of 24.9 grams with 18% less MoI.

Another example, row C, this is perhaps the most practical improvement we can make. Just one lead is relocated closer to the balance rail. This will decrease the MoI by 11% while maintaining the same target front weight of 24.9 grams.

By using these spread sheets, we can see the effect of possible modifications to an action before we make any physical changes.

Using the principles contained in these spread sheets means we can make other MoI adjustments. For example, if a customer asks us to make a touch heavier we could put two additional leads into a key the same distance from balance pin one front and one back, see Fig. 14. This is not new idea, and some manufacturers have already done it to their pianos. Using MoI spread sheet however, you can set the exact amount of additional MoI.

This re-leaded key (Fig. 14) has 93% more MoI than the original key. The key has a much heavier touch feeling than the original key even though DW, BW and UW have not changed. Obviously you can control the effect by putting smaller leads and changing the distance from the balance pin. (See Fig 15)



(Fig. 14) Additional leads into both side of an upright key.

MoI calculating spread sheet		Kawai K30 E (upright)																		
key	Note	Lead (front)		Wood (front)				Key top		Wood (back)					Capstn	Lead (back)		MoI		
#		distance #1 (mm)	mass of lead	160~ 120	120~ 80	80~ 40	40~0	distance (mm)	mass (g)	0~40	40~ 80	80~ 120	120~ 160	160~ 200	distance capstan mm	distance #1 (mm)	mass of lead	front	back	Whole key g cm2
40	C4			5.9	4.6	4.6	5.6	132	8.3	6.3	4.3	4.3	6.4	0.8	144.0			3251	2725	5976
		120	19.4	5.9	4.6	4.6	5.6	132	8.3	6.3	4.3	4.3	6.4	0.8	144.0	120.0	19.4	6044	5519	11563
		additional lead														additional lead				

(Fig. 15) MoI spread sheet of sample upright key with additional leads

You might notice that my MoI spread sheet also contains columns labelled “Center of Torque” and “CoG position ratio” (Fig. 13). I have included these to compare my results with the ratio of the front key length to the center of Gravity, (0.429) that Emerson found to be the most effective position to achieve effective key acceleration (page 26 - 28, April 2013, Piano Technicians Journal). Interestingly the practical placement of key leads I achieved comes closest to Emerson’s value.

Conclusion

I hope you enjoy the benefits of combining adjusting the static touch weight using Stanwood’s equation and adjusting the kinetic touch using these methods. These methods can also be applied to upright pianos.

The Stanwood system has vastly expanded our understanding of touch weight. I hope my article adds to our knowledge of the Moment of Inertia especially when it is coupled with our ever expanding knowledge of balance weight and action geometry.

As my method doesn’t require advanced knowledge of mathematics or physics, I am more than happy to share my spread sheets and I hope that technicians will find them useful in their work.

Here’s to customers who love the feeling of their piano!

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